Two-Step Procedure for Determining Three-Dimensional Conduction Z-Transfer Function Coefficients for Complex Building Envelope Assemblies

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ABSTRACT

A two-step method of derivation of conduction z-transfer function coefficients for three-dimensional wall assemblies is described in this paper. Results of the calculations are presented for clear walls listed in ASHRAE Research Project 1145-TRP and some other two-core masonry block wall assemblies. Overall resistances, three-dimensional response factors, and structural factors have been computed using the finite-difference computer code. The z-transfer function coefficients were then derived from sets of linear equations, which included relationships with the response factors and compatibility conditions. These equations were solved using a minimum error procedure. Very good agreement between heat fluxes calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients was achieved.

Additionally, wall time constants, which are important for hot box test measurements, were developed from the asymptotic values of the response factors' ratios.

INTRODUCTION

Transfer function procedures developed by Stephenson and Mitalas in 1971 pertain to structures made up of layers of homogeneous materials and have no allowance for walls with thermal bridges in which three-dimensional heat flow occurs. Since that time, the z-transfer function method has been used in passive solar energy calculations and whole building energy simulation programs to model one-dimensional heat transfer through walls, roofs, floors, and foundations.

DOE-2 and other older generation whole-building energy calculation tools are utilizing (in their heat transfer calculations) one-dimensional response factors for building envelope assemblies. Completed during 2002, ASHRAE Research Project 1145-TRP, "Modeling Two and Three-Dimensional Heat Transfer through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs" demonstrated inaccuracies related with this simplified one-dimensional approach (ASHRAE 2001). New and more accurate three-dimensional calculation procedures were developed and evaluated at that time. They were based on the theoretical concept of the equivalent wall and transient three-dimensional finite-difference simulations.

Today, several whole-building energy simulation tools including EnergyPlus use conduction z-transfer function coefficients in their energy calculations. Also, most of the building envelope assemblies are not one-dimensional any more. Very often, they represent complex three-dimensional networks of structural and insulation materials. That is why it was so important to develop a relatively simple procedure bridging already existing computational methods utilizing three-dimensional response factors and equivalent wall theory with more advanced methods employing the conduction z-transfer function coefficients.

Seem et al. (1989a, 1989b) presented a method for calculating transfer functions for multidimensional heat transfer from a state space formulation. Spatial discretization of the problem results in a set of first-order differential equations. Exact solution to this set of equations is determined to represent response to the thermal excitation modeled by a continuous, piecewise linear curve.

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Burch et al. (1990) presented a numerical procedure for calculating first-order conduction transfer function coefficients for complex building constructions containing two-dimensional thermal bridges. The heat transfer response to the ramp excitation was predicted by the finite-difference model; then regression analysis was applied to subtract the steady-state response and to determine the first pole of the transfer function.

Brown and Stephenson (1993) developed a method to determine transfer function coefficients from the surface frequency response. This method, based on the Laplace and z-transfer function formalism, has been used to determine the z-transfer functions of the full-scale wall specimens with complex geometries, using guarded hot box procedures.

A method of derivation of conduction z-transfer function coefficients from the response factors for three-dimensional wall assemblies is described in this paper. Response factors, which represent mean surface heat flux generated by triangular temperature excitations at discrete time instants, are used as the input data to determine z-transfer function coefficients from the set of linear equations, which includes relationships with the response factors and compatibility conditions. This infinite set of equations is to be solved applying cut off and using minimum error procedure. The method has already been presented in the ASHRAE 1145-TRP final report (ASHRAE 2001) and in the paper of Kossecka and Kosny (2001) as an alternative to the equivalent wall method. It was applied there to clear walls and selected details. Surface film resistances were neglected at that time. The method gives very good results in the sense that heat flux profiles obtained from simulations, using the response factors and the z-transfer function coefficients, in most cases almost coincide.

In this paper, results of the conduction z-transfer function coefficient calculations are presented for clear walls listed in the ASHRAE 1145-TRP report and several other two-core block masonry walls, including surface film resistances. Overall resistances, three-dimensional response factors, and the so-called structure factors have been computed using the finite-difference computer code. Very good agreement between heat fluxes calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients was achieved.

RELATIONSHIPS BETWEEN RESPONSE FACTORS AND Z-TRANSFER FUNCTION COEFFICIENTS

In terms of the response factors, heat flux across the interior surface of a wall element at time instant $n\delta$, $Q_{i,n\delta}$, can be represented as follows (Kusuda 1969; Clarke 1985):

$$Q_{i,n\delta} = \sum_{k=0}^{n} [X_k T_{i,(n-k)\delta} - Y_k T_{e,(n-k)\delta}] \tag{1}$$

where $\{T_{i,n\delta}\}$ and $\{T_{e,n\delta}\}$ are sequences of the ambient (or surface) temperatures values, and $\{X_n\}$ and $\{Y_n\}$ are sequences of the response factors.

As far as three-dimensional problems are concerned, heat flux values in Equation 1, as well as response factors, are to be understood as averages over the surfaces of a wall element, separated from the rest of the wall by an adiabatic lateral surface. Driving temperatures are functions of time only and do not depend on spatial coordinates, which is also the case when boundary conditions of the first kind are assumed. Dimensions of the element and location of the cut-off surface are to be established while developing a three-dimensional model, in order to determine its thermal characteristics.

The z-transform of the interior heat flux, $Z[Q_i]$ is related to the z-transforms of the interior and exterior temperature, $Z[T_i]$ and $Z[T_e]$, by the following equation (see Jury 1964):

$$Z[Q_i] = Z[\{X_n\}] \cdot Z[T_i] - Z[\{Y_n\}] \cdot Z[T_e] \tag{2}$$

where $Z[\{X_n\}]$ and $Z[\{Y_n\}]$ are the z-transforms of the sequences of the response factors, $\{X_n\}$ and $\{Y_n\}$:

$$Z[Q_i] = \sum_{n=0}^{\infty} Q_{i,n\delta} z^{-n}, \qquad Z[\{X_n\}] = \sum_{n=0}^{\infty} X_n z^{-n}, \qquad Z[\{Y_n\}] = \sum_{n=0}^{\infty} Y_n z^{-n} \tag{3}$$

The compatibility condition response factors X_n and Y_n should satisfy

$$\sum_{n=0}^{\infty} X_n = \sum_{n=0}^{\infty} Y_n = \frac{1}{R}, \tag{4}$$

which is equivalent to the following condition for the z-transforms $Z[\{X_n\}]$ and $Z[\{Y_n\}]$:

$$\lim_{z \to 1} Z[\{X_n\}] = \lim_{z \to 1} Z[\{Y_n\}] = \frac{1}{R} \tag{5}$$

R denotes the overall resistance per unit surface area, determined from the average heat flux in the steady-state conditions.

Now let $Z[\{X_n\}]$ and $Z[\{Y_n\}]$ be given as the quotients

$$Z[\{X_n\}] = \frac{1}{R}\frac{C(z)}{D(z)}, \qquad Z[\{Y_n\}] = \frac{1}{R}\frac{B(z)}{D(z)}, \tag{6}$$

where

$$B(z) = \sum_{n=0}^{\infty} b_n z^{-n}, \qquad C(z) = \sum_{n=0}^{\infty} c_n z^{-n}, \qquad D(z) = \sum_{n=0}^{\infty} d_n z^{-n}. \tag{7}$$

Equation 2 can be rewritten in the form

$$D(z) \cdot Z[Q_i] = \frac{1}{R}\{C(z) \cdot Z[T_i] - B(z) \cdot Z[T_e]\}. \tag{8}$$

Equation 1 for $Q_{i,n\delta}$, assuming $d_0 = 1$, is now replaced by (see Stephenson and Mitalas 1971)

$$Q_{i,n\delta} = \frac{1}{R}\left[\sum_{m=0}^{n} c_m T_{i,(n-m)\delta} - \sum_{m=0}^{n} b_m T_{e,(n-m)\delta}\right] - \sum_{m=1}^{n} d_m Q_{i,(n-m)\delta} \cdot \tag{9}$$

The dimensionless conduction z-transfer function coefficients b_n and c_n correspond to the coefficients b_n and c_n from the *ASHRAE Handbook—Fundamentals* (ASHRAE 1989, 1997) multiplied by R. For the purpose of simulations, only numerically significant coefficients are important.

Equation 6 for the z-transforms, can be rewritten in the following form:

$$C(z) = R \cdot Z[\{X_n\}] \cdot D(z)\ , \qquad B(z) = R \cdot Z[\{Y_n\}] \cdot D(z) \tag{10}$$

They are equivalent to the convolution type relationships between the response factors X_n, Y_n, and the conduction z-transfer function coefficients b_n, c_n, and d_n:

$$b_n = R\sum_{k=0}^{n} Y_{n-k}d_k\ , \qquad c_n = R\sum_{k=0}^{n} X_{n-k}d_k \tag{11}$$

Equation 5 for the z-transforms $Z[\{Y_n\}]$ and $Z[\{X\}]$ now has the following form:

$$\left.\frac{B(z)}{D(z)}\right|_{z=1} = \left.\frac{C(z)}{D(z)}\right|_{z=1} = 1 \tag{12}$$

Equation 12 yields the following compatibility condition for dimensionless z-transfer function coefficients:

$$\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} c_n = \sum_{n=0}^{\infty} d_n \tag{13}$$

DETERMINING THE Z-TRANSFER FUNCTION COEFFICIENTS FROM THE RESPONSE FACTORS

On the basis of Equations 11 and 13, one may try to determine z-transfer function coefficients from the series of response factors Y_n and X_n. This is the most straightforward method; z-transfer functions obtained in this way are expected to exactly reproduce the output for any input function composed of straight-line segments, joining the points that represent its values at $t = n\delta$.

Assuming that z-transfer function coefficients with indices above some n are negligibly small and $d_0 = 1$, we obtain the following set of linear equations:

$$b_0 + b_1 + b_2 + b_3 + \ldots + b_n = 1 + d_1 + d_2 + d_3 + \ldots + d_n \tag{14.1}$$

$$b_0 = RY_0 \tag{14.2}$$

$$b_1 = R(Y_1 + Y_0 d_1) \tag{14.3}$$

$$b_2 = R(Y_2 + Y_1 d_1 + Y_0 d_2) \tag{14.4}$$

$$b_n = R(Y_n + Y_{n-1}d_1 + Y_{n-2}d_2 + Y_{n-3}d_3 + \ldots + Y_0 d_n) \tag{14.n}$$

$$0 = R(Y_{n+1} + Y_n d_1 + Y_{n-1}d_2 + Y_{n-2}d_3 + \ldots + Y_1 d_n) \tag{14.n+1}$$

$$c_0 + c_1 + c_2 + c_3 + \ldots + c_n = 1 + d_1 + d_2 + d_3 + \ldots + d_n \tag{15.1}$$

$$c_0 = RX_0 \tag{15.2}$$

$$c_1 = R(X_1 + X_0 d_1) \tag{15.3}$$

$$c_2 = R(X_2 + X_1 d_1 + X_0 d_2) \tag{15.4}$$

$$c_n = R(X_n + X_{n-1}d_1 + X_{n-2}d_2 + X_{n-3}d_3 + \ldots + X_0 d_n) \tag{15.n}$$

$$0 = R(X_{n+1} + X_n d_1 + X_{n-1}d_2 + X_{n-2}d_3 + \ldots + X_1 d_n) \tag{15.n+1}$$

When structure factors are calculated, together with the resistance and the response factors, one may use conditions imposed by the structure factors on the z-transfer function coefficients as subsidiary equations (Kossecka 1998):

$$\sum_{n=1}^{\infty} nb_n - \sum_{n=1}^{\infty} nd_n = \frac{RC}{\delta}\varphi_{ie}\sum_{n=0}^{\infty} d_n \tag{16}$$

$$\sum_{n=1}^{\infty} nc_n - \sum_{n=1}^{\infty} nd_n = \frac{RC}{\delta}\varphi_{ii}\sum_{n=0}^{\infty} d_n \tag{17}$$

Structure factors φ_{ii} and φ_{ie} are given by

$$\varphi_{ie} = \frac{1}{C}\int_V \rho c_p \theta(1-\theta)dv \tag{18}$$

$$\varphi_{ii} = \frac{1}{C}\int_V \rho c_p (1-\theta)^2 dv \tag{19}$$

where C is the total thermal capacity of the wall element of volume V with an adiabatic lateral surface:

$$C = \int_V \rho c_p dv \tag{20}$$

and θ is the dimensionless temperature for the problem of steady-state heat transfer through this wall element for ambient temperatures $T_i = 0$ and $T_e = 1$. For plane walls, the products $C\varphi_{ii}$, $C\varphi_{ie}$ are equivalent to the thermal mass factors introduced by Anderson (1985; see also ISO 1991).

One may use more equations than the number of unknowns and apply minimum error procedures to get the solution. Maximum indices N_b, N_c, and N_d of the coefficients b_n, c_n, and d_n, which should be included, depend on the specific dynamic thermal properties of a given wall assembly.

In general, the total number of the numerically significant z-transfer function coefficients increases with the resistance and mass of the wall; however, it is not the rule. Trying different kinds of cutoff of the sequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$, one should control the following quantities:

$$E_b = \frac{\sum_{n=0}^{N_c} b_n}{\sum_{n=0}^{N_d} d_n} - 1 , \qquad E_c = \frac{\sum_{n=0}^{N_b} c_n}{\sum_{n=0}^{N_d} d_n} - 1 \tag{21}$$

where E_b and E_c represent resultant errors of the z-transfer function coefficients calculations.

Z-transfer function coefficients determined in this way correspond to the selected time step—here one hour. If a smaller time step is to be used in simulations, say one-half hour or 15 minutes, whole procedure must be repeated.

For plane walls, response factors with sufficiently high indices, above some M, satisfy the condition

$$\frac{Y_{m+1}}{Y_m} = \frac{X_{m+1}}{X_m} = const = \alpha , \qquad m > M \tag{22}$$

$$\alpha = e^{-\frac{\delta}{\tau_1}} \tag{23}$$

where τ_1 is the first, of largest value, time constant of the wall. Therefore, the set of equations (14.1...14.n+k), (15.1...15.n+k) for b_n, c_n, d_n is not infinite in the sense that for sufficiently high indices successive equations are just the preceding ones multiplied by α. Because α is the root of $D(z)$, the condition $d_1 < -\alpha$ is always satisfied.

Response factors for three-dimensional assemblies have, in general, similar properties; however, sometimes their ratios show small variations even for large indices, where they drop several orders of magnitude, as compared with the first ones.

Solving Equations 11 for Y_n, X_n, with $d_0 = 1$, gives the recurrence formulae, which may be used to additionally verify the solution obtained for the z-transfer function coefficients:

$$Y_0 = \frac{b_0}{R} , \qquad X_0 = \frac{c_o}{R} \tag{24}$$

$$Y_n = \frac{b_n}{R} - \sum_{k=1}^{n} Y_{n-k} d_k , \qquad X_n = \frac{c_n}{R} - \sum_{k=1}^{n} X_{n-k} d_k ; \qquad n \geq 1 \tag{25}$$

Results of such verification were presented in the previous paper by Kossecka and Kosny (2001). The method to calculate z-transfer function coefficients presented above appears to be "reversible," in the sense that the response factors recalculated from z-transfer function coefficients are almost the same as the original ones.

CONDUCTION Z-TRANSFER FUNCTION COEFFICIENTS FOR COMMON WALL ASSEMBLIES

Conduction z-transfer function coefficients were calculated for 18 common wall assemblies, including surface film resistances. The list of considered wall assemblies includes uninsulated two-core block masonry wall, two-core block masonry wall with foam insulation inserts, two-core block masonry wall with EPS foam sheathing, concrete sandwich walls with metal and plastic ties, insulated concrete forms (ICF wall), and steel- and wood-framed walls.

Dynamic thermal properties of most of them were analyzed in the frames of the ASHRAE 1145-TRP project, "Modeling Two and Three-Dimensional Heat Transfer Through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs" (ASHRAE 2001; Kossecka and Kosny 2001). However, all calculations were performed at that time for boundary conditions of the first kind, which means that surface film resistances were not included. Drawings of those wall assemblies, with simulation areas dimensioned, are included in the final report of the project. They are also available at ORNL's Internet site (http://www.ornl.gov/sci/roofs+walls/research/detailed_papers/whole_bldg/index.html). Drawings of most common structures may be found in chapter 24 of the *1997 ASHRAE Handbook—Fundamentals*.

Response factors, overall resistances, and structure factors were calculated using the finite difference computer code HEATING 7.2 (Childs 1993) for boundary conditions of the convective type, with standard values of the surface film resistances, $R_i = 0.12$ m²·K/W (0.69 ft²·°F·h/Btu) and $R_e = 0.06$ m²·K/W (0.33 ft²·°F·h/Btu). Thermal characteristics of the wall assemblies: overall resistance, R_u, U-factor, and capacity, C, are presented in Tables 1 and 3.

The conduction z-transfer function coefficients were determined as the approximate solutions of the finite system of equations generated by Equations 14, 15, 16, and 17. The resultant errors, E_b and E_c, were calculated while trying different kinds of the cutoff of the sequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$ to satisfy compatibility Equation 13 as well as possible. Modern professional calculation software allows one to easily examine different solutions of the problem, modifying the numbers of unknown variables and using minimum error procedure to find the solution of a system of N linear equations with M variables ($N \geq M$).

The results are collected in Tables 2 and 4. Accuracy is within five decimal digits; maximum index of a coefficient does not exceed 5. Negative values of the coefficients b_n with higher indices, which appear for all steel- and wood-framed wall assemblies and also concrete blocks, seem questionable at the sight. It was necessary to admit them, to satisfy with sufficient accuracy, compatibility Equation 13. Only for sandwich walls are all b_n positive. For the coefficients c_n and d_n, the sign sequence is always + and –, alternately. One should take into account, however, that negative value of some b_n does not mean that the impact of a temperature value may be "negative," as temperatures enter into the expression for the current heat flux

Table 1. Overall Resistance, U-Factor, Capacity, and Time Constant for Concrete and Insulation Wall Assemblies

No.	Wall Assembly	R_u m²·K/W [°F·ft²·h/Btu]	U W/m²·K [Btu/h ft²·°F]	C kJ/m²·K [Btu/ft²·°F]	τ_s h
1	Empty concrete 60 blocks	0.7383 [4.1946]	1.3546 [0.2384]	128.37 [6.2824]	7.37
2	Insulated concrete 60 blocks	1.2489 [7.0960]	0.8007 [0.1409]	129.25 [6.3253]	6.75
3	Empty concrete 60 blocks + 2.5-cm [1-in] EPS foam outside	1.4758 [8.3851]	0.6776 [0.1193]	129.11 [6.3186]	10.58
4	Empty concrete 60 blocks+ 2.5-cm [1-in] EPS inside	1.4552 [8.3851]	0.6872 [0.1209]	129.11 [6.3186]	9.73
5	Empty concrete 140 blocks	0.3851 [2.1879]	2.5969 [0.4574]	292.82 [14.3304]	5.64
6	Insulated concrete 140 blocks	0.5636 [3.2021]	1.7744 [0.3122]	293.70 [14.3734]	5.77
7	Empty concrete 140 blocks+ 2.5-cm [1-in] EPS inside	1.1212 6.3707	0.8919 [0.1570]	293.57 [14.3667]	12.55
8	Empty concrete 140 blocks+ 2.5-cm [1-in] EPS outside	1.1212 6.3707	0.8919 [0.1570]	293.57 [14.3667]	9.55
9	Sandwich wall with metal ties	1.5395 [8.7474]	0.6495 [0.1143]	301.96 [14.7777]	5.50
10	Sandwich wall with plastic ties	2.0534 [11.6668]	0.4870 [0.0857]	301.73 [14.7663]	5.62
11	ICF – wall	2.1638 [12.2942]	0.4622 [0.0813]	310.10 [15.1757]	34.72

Table 2. Z-Transfer Function Coefficients for Three-Dimensional Models of Concrete and Insulation Wall Assemblies, Including Surface Film Resistances

No.	n	0	1	2	3	4	5	S	E
	b_n	0.01758	0.14672	-0.04052	-0.06436	-0.00296		0.05646	-0.01645
1	c_n	3.20173	-5.97235	3.33807	-0.51873	0.00882	-0.00012	0.05741	0.00000
	d_n	1	-1.51165	0.63364	-0.06458			0.05741	-
	b_n	0.00407	0.05540	0.01669	-0.02796	-0.00244		0.04575	-0.00285
2	c_n	5.40681	-11.41823	7.97783	-2.09542	0.17809	-0.00309	0.04599	0.00244
	d_n	1	-1.737821	0.97536	-0.20345	0.01179		0.04588	-
	b_n	0.00414	0.05219	-0.00955	-0.03183	0.00431	0.00090	0.02016	-0.03194
3	c_n	6.40117	-15.23480	12.55344	-4.11490	0.41694	-0.00102	0.02083	0.00000
	d_n	1	-2.02634	1.37217	-0.34733	0.02232		0.02083	-
	b_n	0.00582	0.06751	-0.00565	-0.03785	-0.00238		0.02746	-0.02136
4	c_n	1.62339	-3.07361	1.77296	-0.30498	0.01030		0.02806	0.00000
	d_n	1	-1.75990	0.90918	-0.12136	0.00013		0.02806	-
	b_n	0.01570	0.07061	-0.03852	-0.00504	0.00481	0.00002	0.04758	-0.00007
5	c_n	2.48430	-4.98476	3.36797	-0.89864	0.07913	-0.00038	0.04762	0.00092
	d_n	1	-1.81134	1.10652	-0.26870	0.02111		0.04758	-
	b_n	0.00270	0.02360	0.00735	0.00114	0.00185	-0.00008	0.03656	0.00000
6	c_n	3.62858	-7.84323	5.79526	-1.70882	0.16559	-0.00082	0.03656	0.00000
	d_n	1	-1.95117	1.30987	-0.35315	0.03101		0.03656	-
	b_n	0.00398	0.02456	-0.00633	-0.00405	0.00147	0.00021	0.01986	-0.00859
7	c_n	7.23373	-15.34183	10.65019	-2.67216	0.15025		0.02003	0.00000
	d_n	1	-1.92550	1.20168	-0.26899	0.011283		0.02003	-
	b_n	0.00651	0.03946	0.00180	-0.00062	0.00085	-0.00021	0.04780	-0.01066
8	c_n	1.31599	-2.08366	1.04018	-0.30907	0.08794	-0.00305	0.04834	0.00050
	d_n	1	-1.50948	0.70220	-0.20029	0.05589		0.04831	-
	b_n	0.00058	0.01539	0.02438	0.00428			0.04463	0.00000
9	c_n	16.91909	-31.65908	15.63805	-0.86072	0.00729		0.04463	0.00000
	d_n	1	-1.58408	0.64365	-0.01494			0.04463	-
	b_n	0.00034	0.01289	0.02440	0.00509	0.00007		0.04279	0.00000
10	c_n	22.52628	-42.30895	20.91198	-1.09482	0.00830		0.04279	0.00000
	d_n	1	-1.59168	0.64833	-0.01386			0.04279	-
	b_n	0.00016	0.00314	0.00522	0.00117	-0.00012	-0.00006	0.00952	-0.02542
11	c_n	12.02566	-27.94417	21.07057	-5.51797	0.37568		0.00977	0.00000
	d_n	1	-1.84879	1.08007	-0.23645	0.01494		0.00977	-

Table 3. Overall Resistance, U-Factor, Capacity, and Time Constant for Steel Stud and Wood Stud Wall Assemblies, Completed with Fiberglass Insulation

No.	Wall Assembly	R_u $m^2 \cdot K/W$ [°F·ft²·h/Btu]	U W/m²·K [Btu/h ft²·°F]	C kJ/m²·K [Btu/ ft²·°F]	τ_s h
1	8.9 cm [3.5 in.] steel studs	1.8137 10.3049	0.5514 [0.0970]	30.53 [1.4942]	0.92
2	8.9 cm [3.5 in.] steel studs + 2.5 cm [1 in.] +brick	2.4657 [14.0097]	0.4056 [0.0714]	148.68 [7.2760]	3.47
3	14 cm [5.5 in.] steel studs	2.3016 [13.0770]	0.4345 [0.0765]	31.45 [1.5389]	1.02
4	14 cm [5.5 in.] steel studs + 2.5 cm [1 in.] + stucco	2.9165 [16.5711]	0.3429 [0.0603]	60.26 [2.9489]	1.67
5	14 cm [5.5 in.] steel studs + 2.5 cm [1 in.] + brick	2.9453 [16.7345]	0.3395 [0.0598]	182.81 [8.9465]	6.41
6	8.9 cm [3.5 in.] wood studs	2.1918 [12.4534]	0.4563 [0.0803]	36.64 [1.7933]	2.00
7	14 cm [5.5 in.] wood studs	3.2605 [18.5255]	0.3067 [0.0540]	37.43 [1.8317]	3.37

Table 4. Z-Transfer Function Coefficients for Three-Dimensional Models of Steel Stud and Wood Stud Wall Assemblies, Including Surface Film Resistances

No.	n	0	1	2	3	4	5	Σ	E
12	b_n	0.10290	0.38353	-0.03155	-0.02538	0.00026		0.42975	-0.00324
	c_n	6.18416	-8.32412	2.85936	-0.29637	0.00811		0.43115	0.00000
	d_n	1	-0.71425	0.15485	-0.00949	0.00003		0.43115	-
13	b_n	0.00097	0.03435	0.05699	-0.00303	-0.00477	-0.00014	0.08436	-0.01025
	c_n	7.39937	-17.21008	14.15948	-4.85369	0.59188	-0.00123	0.08572	0.00563
	d_n	1	-1.61537	0.86672	-0.17609	0.00998		0.08524	-
14	b_n	0.06863	0.35386	0.00284	-0.03287	-0.00004		0.39241	-0.00390
	c_n	7.84153	-11.13393	4.14433	-0.47195	0.01396		0.39394	0.00000
	d_n	1	-0.77786	0.18438	-0.01261	0.00005		0.39394	-
15	b_n	0.01392	0.15796	0.08631	-0.02284	-0.00293	0.00002	0.23245	-0.00878
	c_n	8.70566	-16.10892	9.75526	-2.28619	0.17089	-0.00038	0.23632	0.00770
	d_n	1	-1.11905	0.40682	-0.05578	0.00253		0.23451	-
16	b_n	0.00048	0.01898	0.03344	-0.00107	-0.00267	-0.00005	0.04911	-0.01959
	c_n	10.09872	-25.36201	22.57858	-8.36114	1.10193	-0.00571	0.05037	0.00574
	d_n	1	-1.69548	0.90411	-0.16292	0.00437		0.05009	-
17	b_n	0.07152	0.25529	-0.06834	-0.02065	0.00262		0.24044	-0.00326
	c_n	7.49542	-12.34863	5.84959	-0.77689	0.02174		0.24123	0.00000
	d_n	1	-1.03256	0.29774	-0.02395			0.24123	-
18	b_n	0.05121	0.22681	-0.17619	-0.01033	0.01910		0.11059	-0.00897
	c_n	11.07512	-23.55336	16.88365	-4.85108	0.55727		0.11159	0.00000
	d_n	1	-1.47720	0.72559	-0.14766	0.01087		0.11159	-

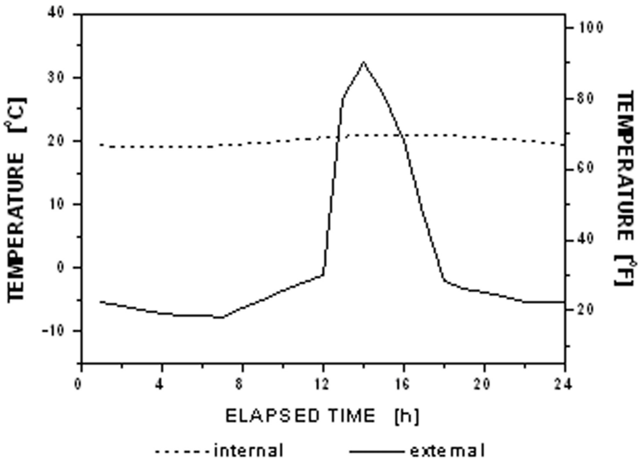

Figure 1 *Internal and external temperature courses used for simulations.*

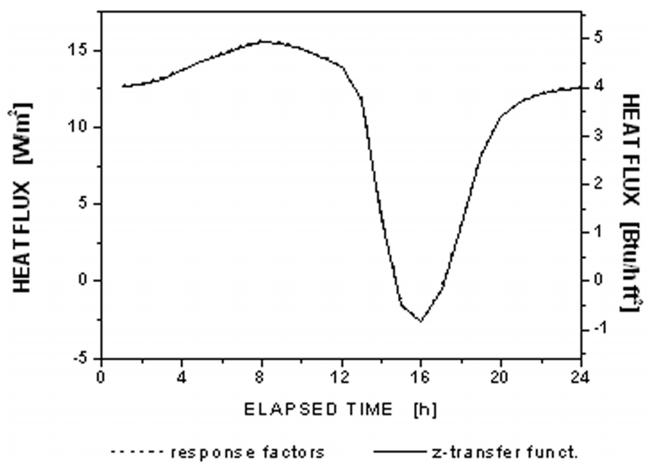

Figure 2 *Comparison of the heat flux simulation results for the 8.9-cm [3.5-in.] steel stud wall.*

(Equation 9) not only through b_n and c_n, but also through the preceding values of the heat flux itself.

ESTIMATION OF THE WALLS TIME CONSTANTS

Proper estimation of a wall's time constant is important for the hot box test measurements. The hot box apparatus is designed to determine thermal performance for representative test specimens by establishing and maintaining a desired steady temperature difference across the specimen for the period of time necessary to ensure constant energy flux and steady temperatures and for an additional period adequate to measure these quantities to the desired accuracy. The required time to reach stability for a steady-state test depends upon the properties of both the specimen and of the apparatus, as well as upon the initial and final conditions of the test. A combined apparatus and specimen time constant calculated from dimensions and estimated physical properties can be used in estimating stabilization time. Since the test apparatus does not change with test sample, it is recommended that the apparatus time constant be determined by experimental means (see ASTM Standard C 1363-97). Repeating the same experimental procedure for every test sample would be expensive, so a calculation method to estimate the sample time constant might be useful.

Assuming the analogy between one-dimensional and three-dimensional cases means that the asymptotic transient response decay is also governed by a single exponential function, we may, therefore, determine the largest wall's time constant, τ_s, from the ratio, α, of the response factors with high indices (Equation 22):

$$\tau_s = -\frac{\delta}{\ln\alpha} \tag{26}$$

Values of time constants developed from the asymptotic values of the response factors' ratios are collected in Tables 1 and 3. They were calculated to illustrate dynamic thermal properties of walls. In general, time constants calculated in this way for complex wall assemblies, in which three-dimensional heat flow occurs, have higher values than for similar (in the sense of total resistance, capacity, and arrangement of materials with different thermal properties) plane-layer wall assemblies.

Test Simulations

Test simulations were performed, using as the external temperature excitation the sol-air temperature[1] calculated for a vertical surface facing west, for a sunny day of February in Warsaw (see Figure 1). Internal temperature represented periodic variations with amplitude of 1°C (1.8°F) around a mean value of 20°C (68°F). The same daily temperature courses were repeated several times, to eliminate the effect of initial conditions. The heat flux across the inside surface of a wall was calculated in two ways, using response factors for the three-dimensional model and three-dimensional z-transfer function coefficients derived from the response factors.

Results of simulations for the lightweight 3.5 steel stud wall, of time constant below one hour, are presented in Figure 2. Differences between the heat flux values calculated using

1. Sol-air temperature is the equivalent outdoor temperature that will cause the same rate of heat flow at the surface and the same temperature distribution through the material as the current outdoor air temperature, the solar gains on the surface, and the net radiant exchange between the surface and its environment.

$$T_s = T_o + G{\cdot}a{\cdot}R_{so}$$

where
T_s = sol-air temperature (°C),
T_o = outside air temperature (°C),
G = total incident solar radiation (W/m²),
a = solar absorbance of surface (0-1), and
R_{so} = outside air-film resistance.

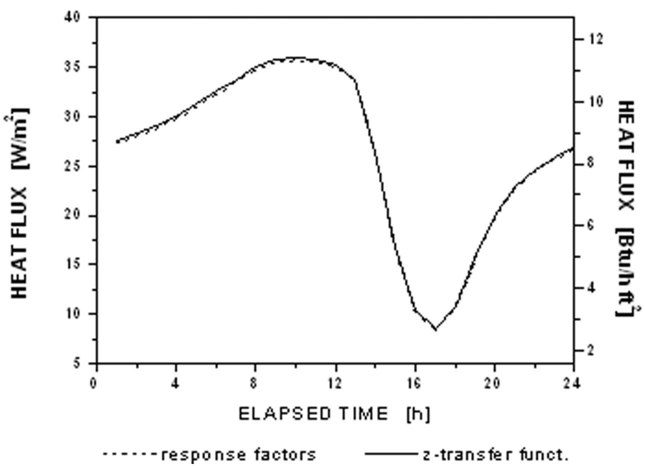

Figure 3 *Comparison of the heat flux simulation results for the empty concrete 60 blocks.*

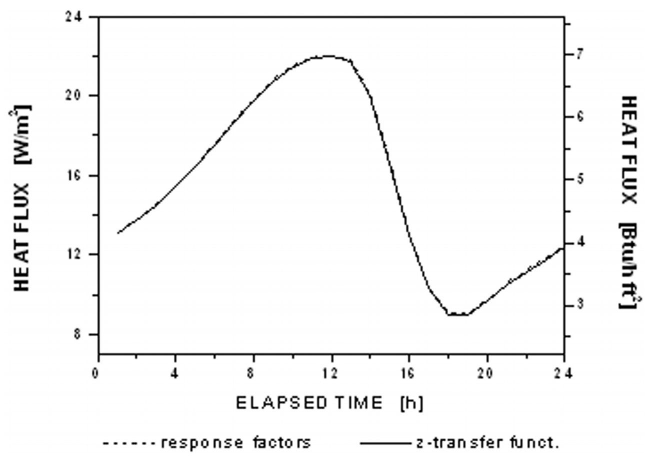

Figure 4 *Comparison of the heat flux simulation results for the insulated concrete 60 blocks.*

three-dimensional response factors and three-dimensional z-transfer function coefficients are almost invisible.

Figures 3, 4, and 5 present results of simulations for the concrete and insulated wall assemblies, with time constants between 5 and 10 hours: empty and insulated lightweight concrete blocks and empty heavyweight concrete blocks with 1-in. EPS foam layer inside; differences here are also very small.

Figures 6 and 7 present heat flux profiles for the sandwich wall with metal and plastic ties, respectively. Differences are significant only for the last case. Figure 6 is completed with the steady-state and average steady-state values to show dynamic effects. Steady-state values are to be understood as products of U-factor and ambient temperature differences. Weak internal periodic thermal excitation generates heat flux of significant amplitude, which is a consequence of high internal admittance amplitude of a wall with an internal heavy-weight concrete layer. Average heat flux values perfectly coincide.

When evaluating results of the heat flux simulations, performed using the z-transfer function coefficients calculated from the response factors, one should remember that some errors are generated when calculating the response factors themselves, as Equation 4 is not always satisfied with the required accuracy.

CONCLUSIONS

The method of derivation of the conduction z-transfer function coefficients from the response factors, for three-dimensional wall assemblies including surface film resistances, gives satisfactory results.

The list of 18 wall assemblies considered includes two-core block masonry walls, empty and with insulation inserts, two-core block masonry wall with EPS foam sheathing, concrete sandwich walls with metal and plastic ties, insulated concrete forms (ICF wall), steel- and wood- framed walls.

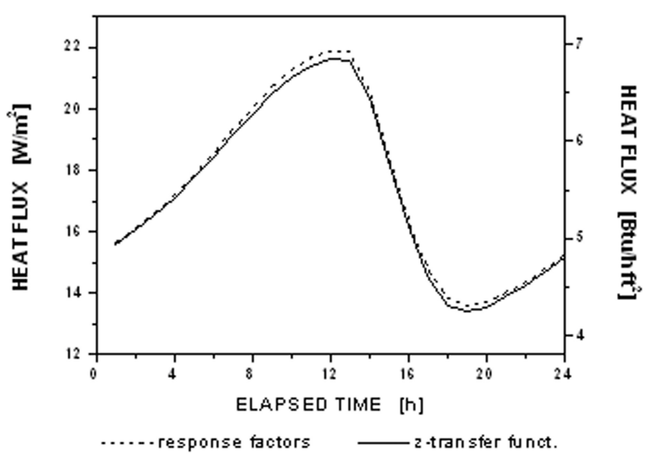

Figure 5 *Comparison of the heat flux simulation results for the empty concrete 140 blocks with +2.5-cm [1-in] EPS foam layer inside.*

Response factors for three-dimensional models, calculated with the help of the finite difference computer code HEATING 7.2, for convective-type boundary conditions, were used as input data to determine z-transfer function coefficients from the primarily infinite set of linear equations, which includes relationships with the response factors and compatibility conditions. For each case, different kinds of cutoff of the sequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$ were considered, and minimum error procedure was applied while seeking for the solutions to satisfy, as best as possible, compatibility conditions.

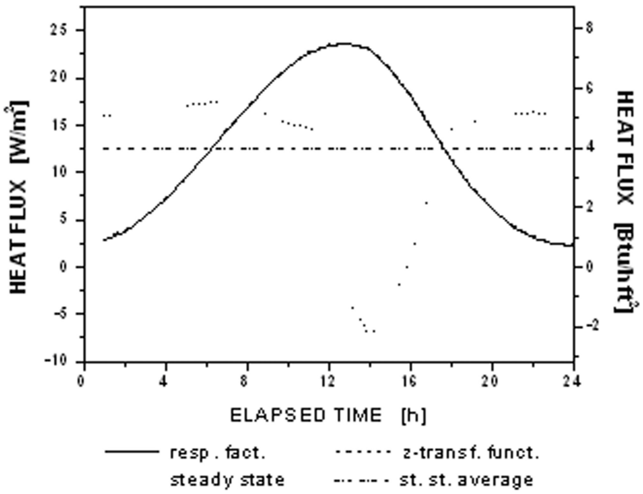

Figure 6 *Comparison of the heat flux simulation results for the sandwich wall with metal ties.*

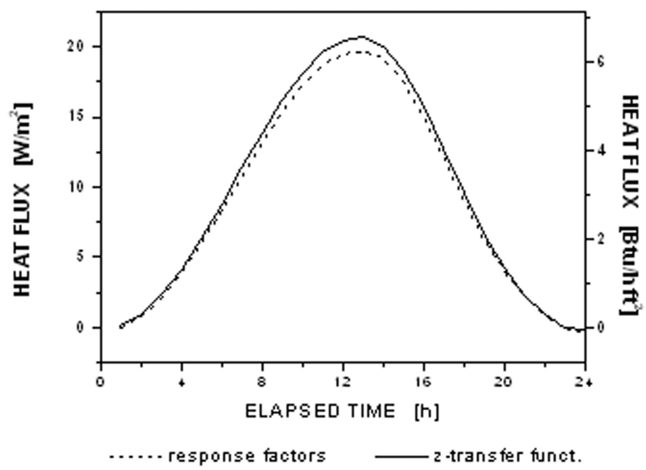

Figure 7 *Comparison of the heat flux simulation results for the sandwich wall with plastic tie.*

The maximum index of a z-transfer function coefficient does not exceed 5 to maintain accuracy of 5 decimal digits. It was necessary to admit negative values of the coefficients b_n with higher indices to satisfy, with sufficient accuracy, compatibility equations. For the coefficients c_n and d_n the sign sequence is always + and –, alternately.

Test simulations, performed for an external temperature excitation of high amplitude, show very good compatibility of the heat flux calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients derived from the response factors.

NOMENCLATURE

δ	=	time instant; (h)
$Q_{i,n\delta}$	=	heat flux at time $n\delta$ across the interior surface; W/m^2 (Btu/(h·ft^2))
$T_{i,n\delta}$	=	interior temperature at time $n\delta$; °C (°F)
$T_{e,n\delta}$	=	exterior temperature at time $n\delta$; °C (°F)
X_n, Y_n	=	response factors; W/(m^2·K) (Btu/(h·ft^2·°F))
$Z\{Q\}$, $Z\{T\}$, $Z\{X\}$, $Z\{Y\}$, $B(z)$, $C(z)$, $D(z)$ = z-transforms		
b_n, c_n, d_n	=	dimensionless heat conduction z-transfer function coefficients (-)
N_b, N_c, N_d	=	maximum index of numerically significant coefficient b_n, c_n, d_n, respectively
E_b, E_c	=	relative errors of the z-transfer function calculations [-]
V	=	volume of a wall element; m^3 (ft^3)
R_u	=	overall thermal resistance per unit surface area of a wall, m^2·K/W (ft^2·°F·h/Btu)
C	=	capacity per unit surface area of a wall, kJ/(m^2·K) (Btu/(ft^2·°F))
c_p	=	specific heat, J/(m^3·K) (Btu/(lb·°F))
ρ	=	density, kg/m^3 (lb/ft^3)
θ	=	dimensionless temperature (-)
φ_{ii}, φ_{ie}	=	structure factors (-)
τ_s	=	largest time constant of a wall

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